ตัวอย่าง: (กลางๆ) ลองหาคำตอบ

① $\int \sin^3 x \cos x \, dx$

วิธีทำ. กำหนดให้ $u = \sin x$

จะได้ $\frac{du}{dx} = \frac{d[\sin x]}{dx} = \cos x$

$\Rightarrow \quad dx = \frac{du}{\cos x}$

พิจารณา $\int \sin^3 x \cos x \, dx = \int u^3 \cancel{\cos x} \frac{du}{\cancel{\cos x}}$

$= \int u^3 \, du$

$= \frac{u^4}{4} + C = \frac{\sin^4 x}{4} + C$

② $\int t^4 \left(\sqrt[3]{3-5t^5}\right) dt$

วิธีทำ. กำหนดให้ $u = 3 - 5t^5$

จะได้ $\frac{du}{dt} = -25t^4 \Rightarrow dt = \frac{du}{(-25)t^4}$

พิจารณา $\int t^4 \left(\sqrt[3]{3-5t^5}\right) dt = \int \cancel{t^4} \left(\sqrt[3]{u}\right) \frac{du}{(-25)\cancel{t^4}}$

$$= \left(\frac{1}{-25}\right)\int u^{1/3}\,du$$

$$= \left(-\frac{1}{25}\right)\frac{u^{4/3}}{\left(4/3\right)} + C$$

$$= \frac{-3}{100}\left(3-5t^5\right)^{4/3} + C$$

③ $\displaystyle\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

<u>วิธีทำ</u> กำหนดให้ $u = e^x$

จะได้ $\frac{du}{dx} = e^x \Rightarrow dx = \frac{du}{e^x}$

ดังนั้น $\displaystyle\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = \int \frac{\cancel{u}}{\sqrt{1-u^2}}\frac{du}{\cancel{u}}$

$$= \int \frac{1}{\sqrt{1-u^2}}\,du$$

$$= \arcsin u + C$$

$$= \arcsin(e^x) + C$$

$\square$

<u>ตัวอย่าง</u>: (ไม่ง่าย) ลองหาคำตอบ

① $\displaystyle\int x^2\sqrt{x-1}\,dx$

วิธีทำ. กำหนดให้ $\boxed{u = x-1} \Rightarrow x = u+1 \Rightarrow x^2 = (u+1)^2$ ✓

จะได้ $\frac{du}{dx} = 1 \Rightarrow dx = du$

พิจารณา $\int x^2\sqrt{x-1}\,dx = \int x^2\sqrt{u}\,du$

$$= \int (u+1)^2 \cdot u^{\frac{1}{2}}\,du$$

$$= \int (u^2+2u+1)\,u^{\frac{1}{2}}\,du$$

$$= \int \left(u^{5/2} + 2u^{3/2} + u^{1/2}\right)du$$

$$= \frac{u^{7/2}}{(7/2)} + \frac{2u^{5/2}}{(5/2)} + \frac{u^{3/2}}{(3/2)} + C$$

$$= \frac{2(x-1)^{7/2}}{7} + \frac{4(x-1)^{5/2}}{5} + \frac{2(x-1)^{3/2}}{3} + C$$

@ $\int \frac{\sin\left(\frac{1}{x}\right)}{3x^2}\,dx$

วิธีทำ. กำหนดให้ $u = 3x^2 \Rightarrow x^2 = \frac{u}{3} \Rightarrow x = \sqrt{\frac{|u|}{3}}$

จะได้ว่า $\frac{du}{dx} = 6x \Rightarrow dx = \frac{du}{6x}$

พิจารณา $\int \frac{\sin\left(\frac{1}{x}\right)}{3x^2}\,dx = \int \frac{\sin\left(\frac{1}{x}\right)}{u}\,\frac{du}{6x} \Rightarrow ?$

กำหนดให้ $u = \frac{1}{x}$

จะได้ $\frac{du}{dx} = \frac{d[x^{-1}]}{dx} = -x^{-2} \Rightarrow dx = \frac{du}{(-x^{-2})}$

$\Rightarrow dx = -x^{2} du$

พิจารณา $\int \frac{\sin\left(\frac{1}{x}\right)}{3x^2} dx = \int \frac{\sin(u)}{3\cancel{x^2}} (-\cancel{x^2} du)$

$= \left(-\frac{1}{3}\right) \int \sin u \, du$

$= \left(-\frac{1}{3}\right)(-\cos u) + C$

$= \frac{1}{3} \cos\left(\frac{1}{x}\right) + C$

③ $\int x^3 e^{x^4} dx$

วิธีทำ กำหนดให้ $u = x^4$

จะได้ $\frac{du}{dx} = 4x^3 \Rightarrow dx = \frac{du}{4x^3}$

พิจารณา $\int x^3 e^{x^4} dx = \int \cancel{x^3} e^{u} \frac{du}{4\cancel{x^3}}$

$= \frac{1}{4} \int e^{u} du$

$= \frac{e^{u}}{4} + C = \frac{e^{x^4}}{4} + C$

④ $\int \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}\,dx$

<u>วิธีทำ</u>. กำหนดให้ $u = e^{x}-e^{-x}$

จะได้ $\frac{du}{dx} = e^{x}+e^{-x} \Rightarrow dx = \frac{du}{e^{x}+e^{-x}}$

ดังนั้น $\int \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}\,dx = \int \frac{\cancel{e^{x}+e^{-x}}}{u}\,\frac{du}{\cancel{e^{x}+e^{-x}}}$

$= \int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \ln|e^{x}-e^{-x}| + C$

⑤ $\int [\sin(\sin\theta)]\cos\theta\,d\theta$

<u>วิธีทำ</u> กำหนด $u = \sin\theta$

จะได้ $\frac{du}{d\theta} = \cos\theta \Rightarrow d\theta = \frac{du}{\cos\theta}$

ดังนั้น $\int [\sin(\sin\theta)]\cos\theta\,d\theta = \int \sin(u)\cancel{\cos\theta}\,\frac{du}{\cancel{\cos\theta}}$

$= \int \sin u\,du$

$= -\cos u + C$

$= -\cos(\sin\theta) + C$

⑥ $\displaystyle\int \frac{dx}{\sqrt{8x-x^2}}$

วิธีทำ. พิจารณา $8x-x^2 = -(x^2-8x)$

$$= -(x^2-8x+16-16)$$
$$= -(x^2-8x+16)+16$$
$$= 4^2-(x-4)^2$$

ดังนั้น $\displaystyle\int \frac{dx}{\sqrt{8x-x^2}} = \int \frac{1}{\sqrt{4^2-(x-4)^2}}\,dx$

กำหนดให้ $u = x-4 \Rightarrow \frac{du}{dx} = 1 \Rightarrow dx = du$

$$\int \frac{1}{\sqrt{8x-x^2}}\,dx = \int \frac{1}{\sqrt{4^2-(x-4)^2}}\,dx$$
$$= \int \frac{1}{\sqrt{4^2-u^2}}\,du$$
$$= \arcsin \frac{u}{4} + C$$
$$= \arcsin \frac{(x-4)}{4} + C$$

⑦ $\displaystyle\int (\sec x + \tan x)^2\,dx$

วิธีทำ พิจารณา

$(\sec x+\tan x)^2 = \sec^2 x + 2\sec x\tan x + \tan^2 x$

Note!

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \Rightarrow \tan^2 x + 1 = \sec^2 x$$

$$\Rightarrow \tan^2 x = \sec^2 x - 1$$

พิจารณา

$$(\sec x + \tan x)^2 = \sec^2 x + 2\sec x \tan x + \sec^2 x - 1$$

$$= 2\sec^2 x + 2\sec x \tan x - 1$$

ดังนั้น

$$\int (\sec x + \tan x)^2 \, dx = \int (2\sec^2 x + 2\sec x \tan x - 1) \, dx$$

$$= 2\tan x + 2\sec x - x + C$$

□